Quantum Walking in Curved Spacetime

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Joint work with Pablo Arrighi and Stefano Facchini.

[P. Arrighi, S. Facchini, MF, *"Quantum walking in curved spacetime."* Quantum Information Processing *(2015): 1-20*]

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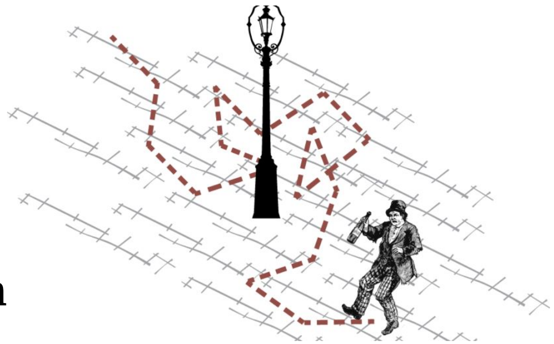
Quantum walks

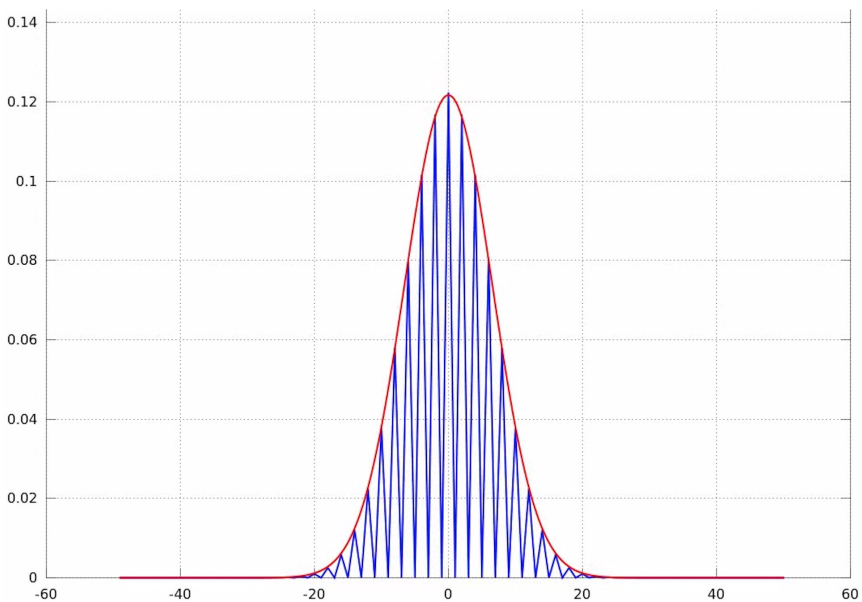
"Hadamard walk" $P = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 \end{pmatrix}$

$Q = \begin{pmatrix} 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$



Classical Random

$$Pr(t+1,x) = \frac{1}{2}Pr(t,x-1) + \frac{1}{2}Pr(t,x+1)$$

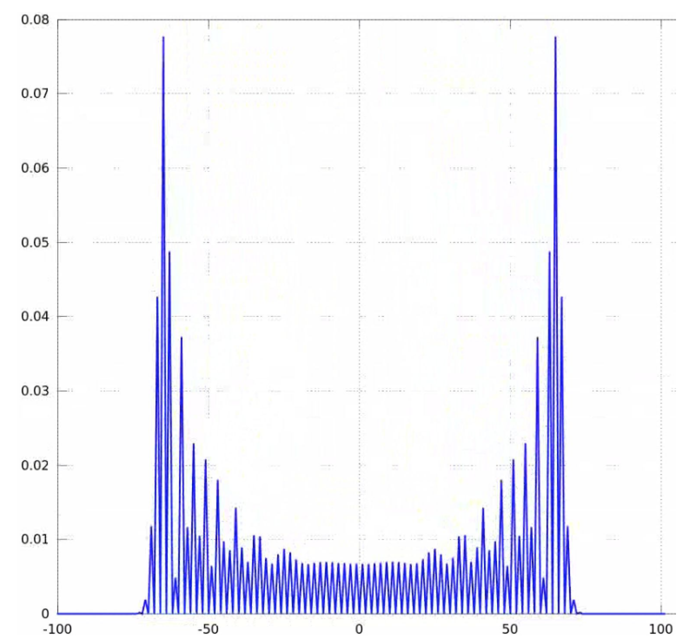


$\sigma(t) \propto \sqrt{t}$ (diffusive)

Quantum walk

$$\psi(t+1,x) = P\psi(t,x-1) + Q\psi(t,x+1),$$

$P + Q = C$ Unitary $\Longrightarrow$ preserves probability

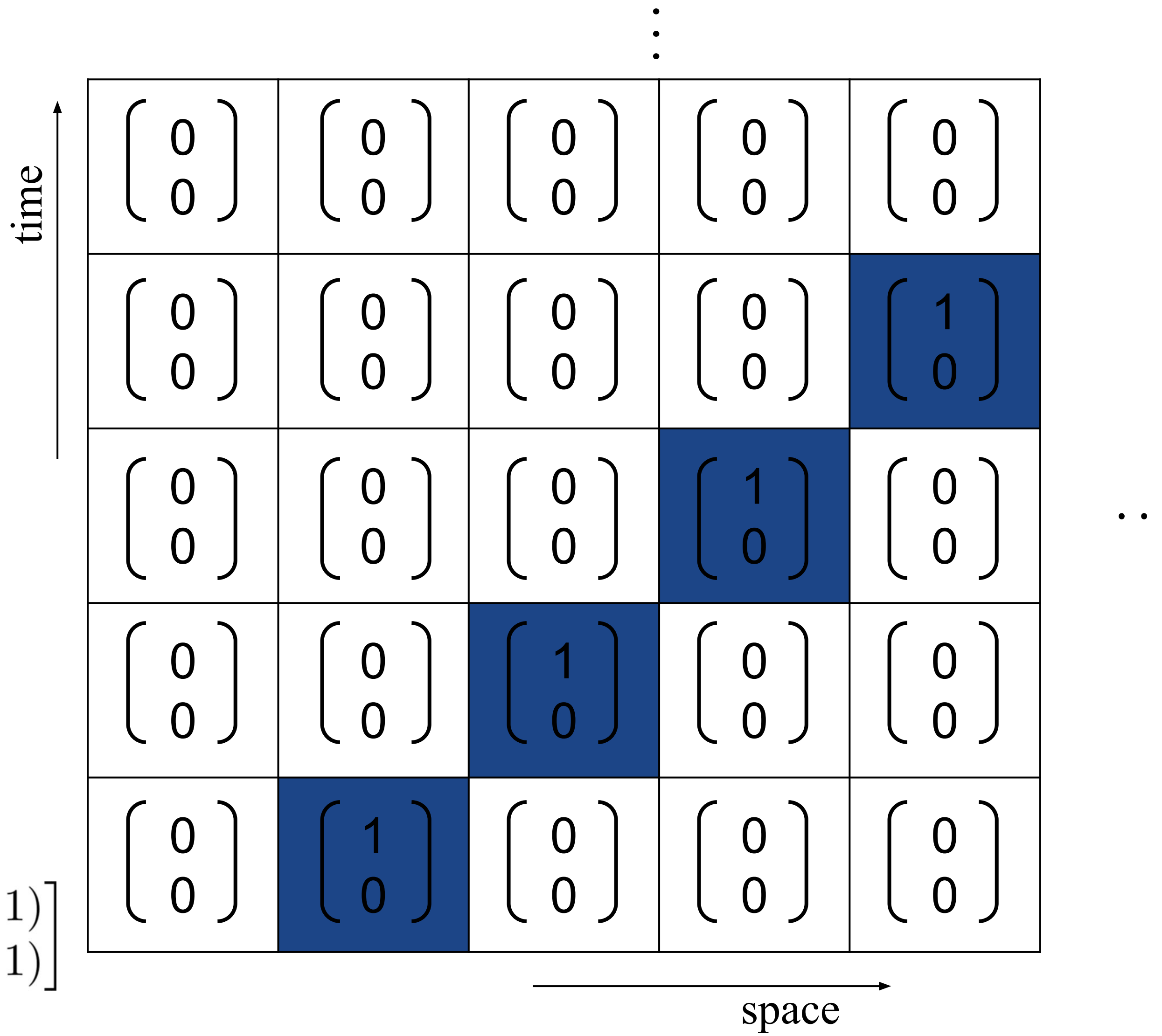


$\sigma(t) \propto t$ (ballistic)

Weyl automaton

Chess game for a neutrino

To the right



$$(t, x) \mapsto \psi(t, x) = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}$$

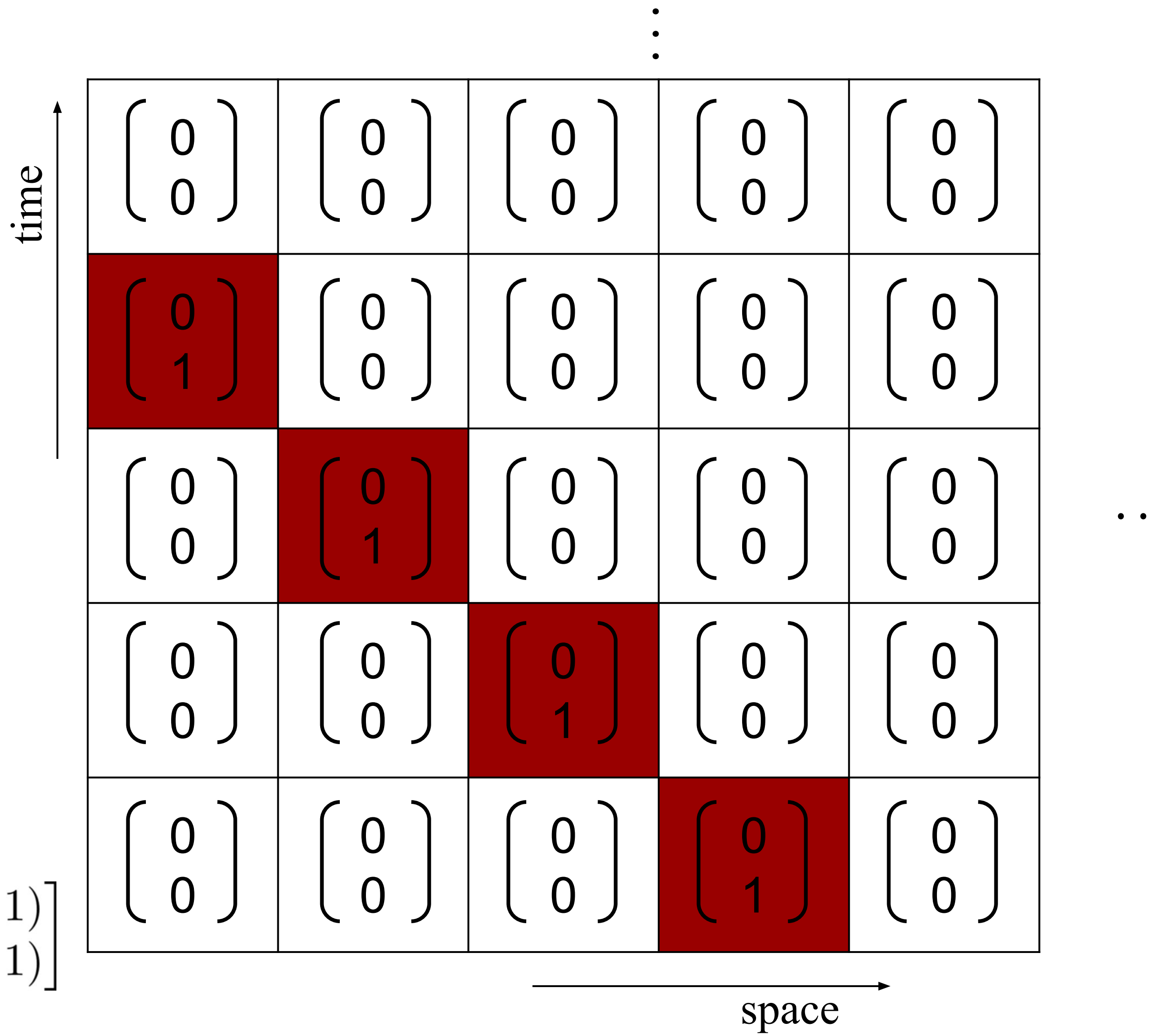
$$\begin{bmatrix} \psi^+(t+1, x) \\ \psi^-(t+1, x) \end{bmatrix} = \begin{bmatrix} \psi^+(t, x-1) \\ \psi^-(t, x+1) \end{bmatrix}$$

Weyl automaton

Chess game for a neutrino

To the right

To the left



$$(t,x) \mapsto \psi(t,x) = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}$$

$$\begin{bmatrix} \psi^+(t+1,x) \\ \psi^-(t+1,x) \end{bmatrix} = \begin{bmatrix} \psi^+(t,x-1) \\ \psi^-(t,x+1) \end{bmatrix}$$

Weyl automaton

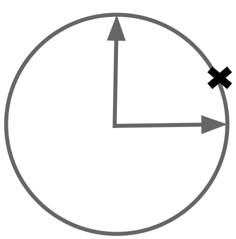
Chess game for a neutrino

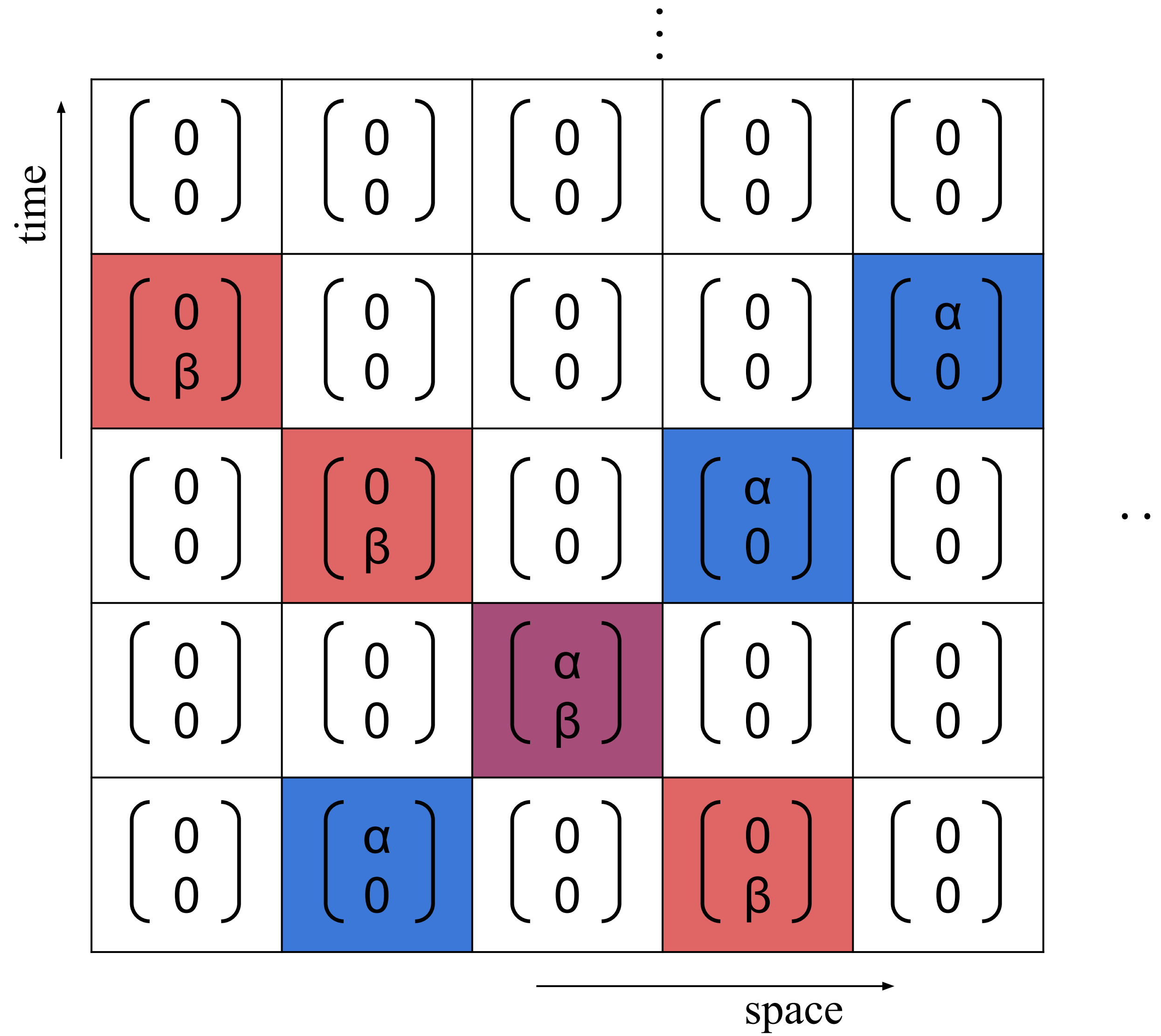
To the right

To the left

Amplitudes:

$|\alpha|^2+|\beta|^2=1$





Weyl automaton

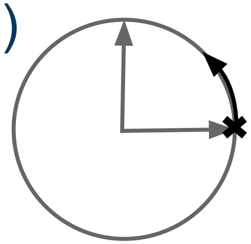
Chess game for a neutrino

Rotations

$$C = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$c = \cos(\theta)$

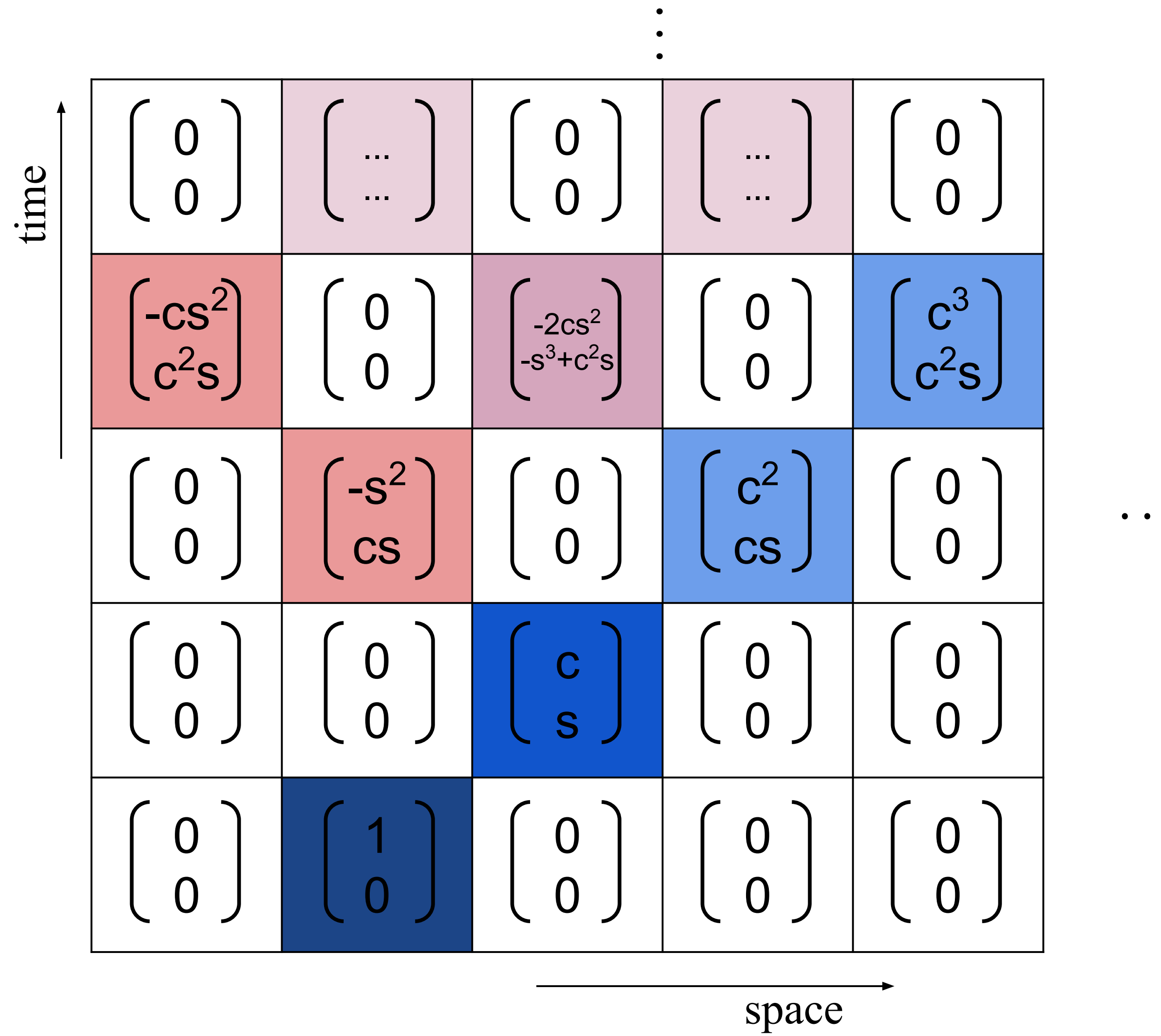
$s = \sin(\theta)$



$\theta = m.\varepsilon$

m = mass

ε = step



Dirac QW

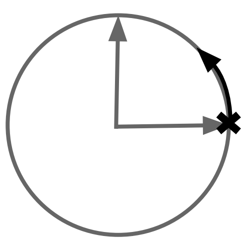
Chess game for an electron

Rotations

$$C = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$c = \cos(\theta)$

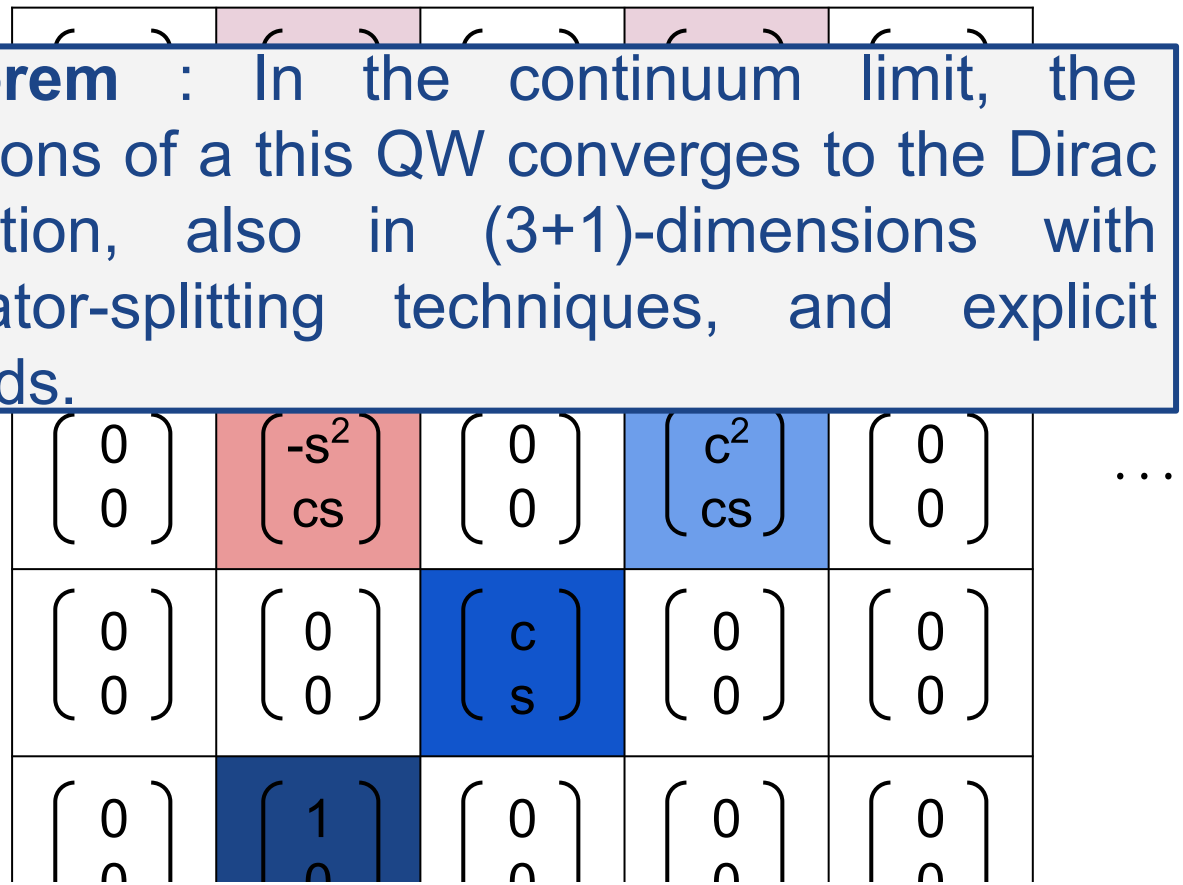
$s = \sin(\theta)$



$\theta = m.\varepsilon$

m = mass

ε = step



Theorem : In the continuum limit, the solutions of a this QW converges to the Dirac Equation, also in (3+1)-dimensions with operator-splitting techniques, and explicit bounds.

[Arrighi, P., Nesme, V., & MF (2014). The Dirac equation as a quantum walk: higher dimensions, observational convergence. *Journal of Physics A: Mathematical and Theoretical*, *47*(46), 465302.]

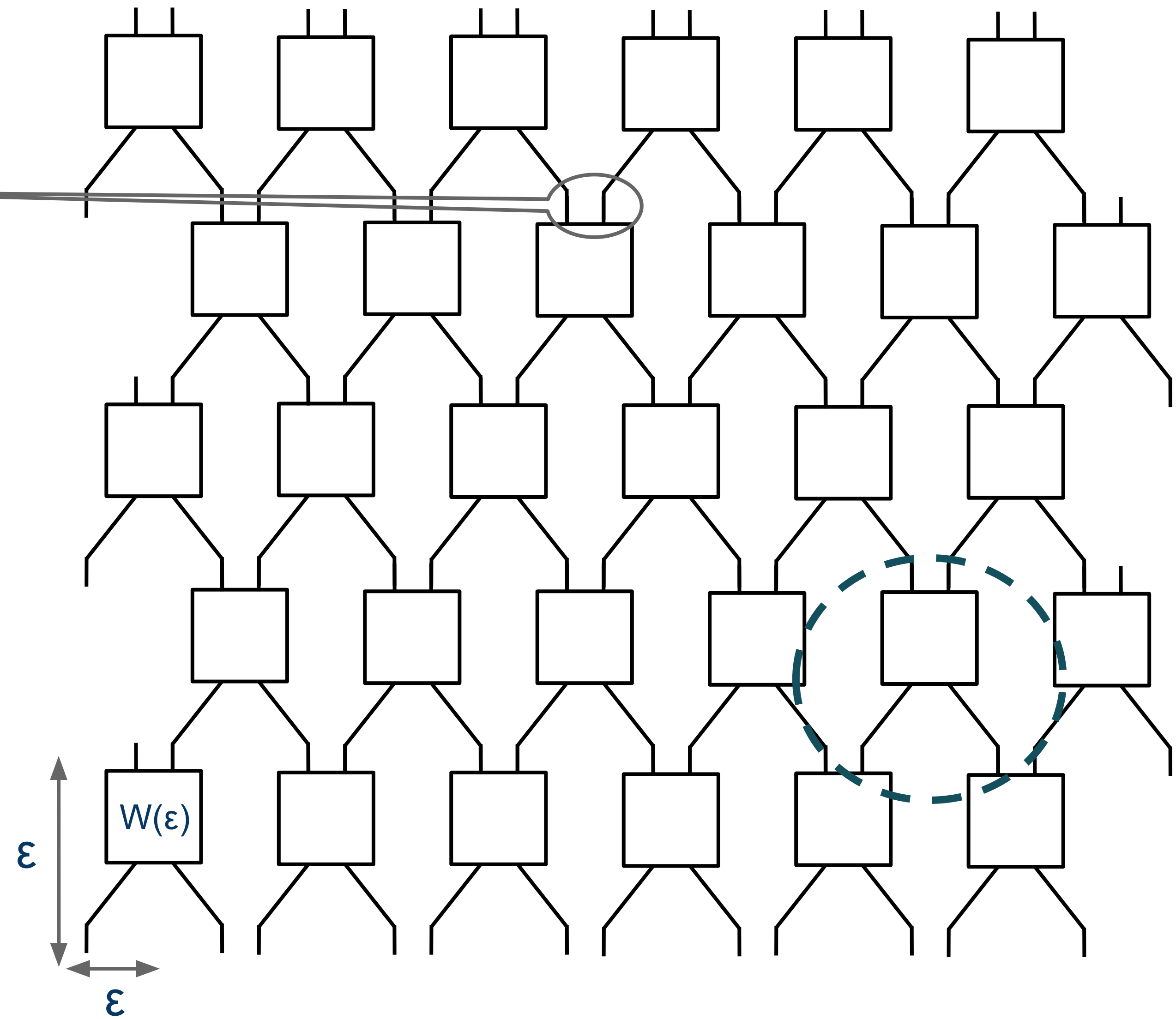
Dirac QW

proof sketch

States:

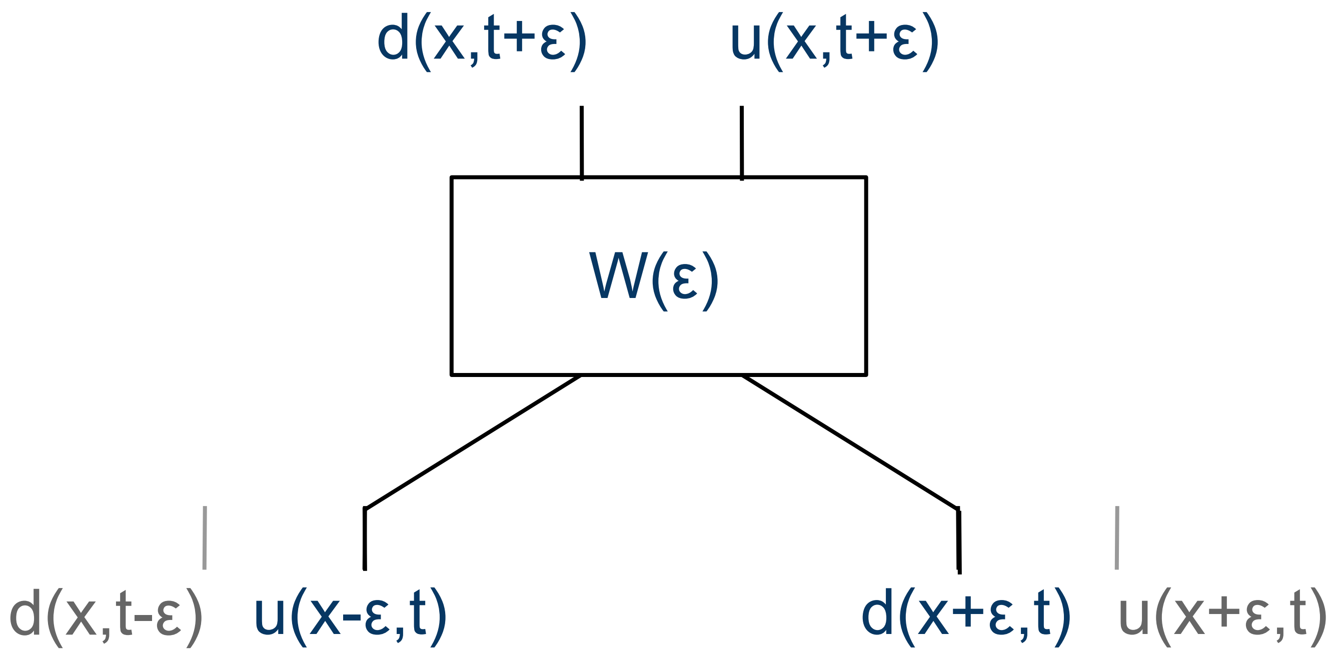
$$\begin{pmatrix} u \\ d \end{pmatrix}$$

Scale: ε



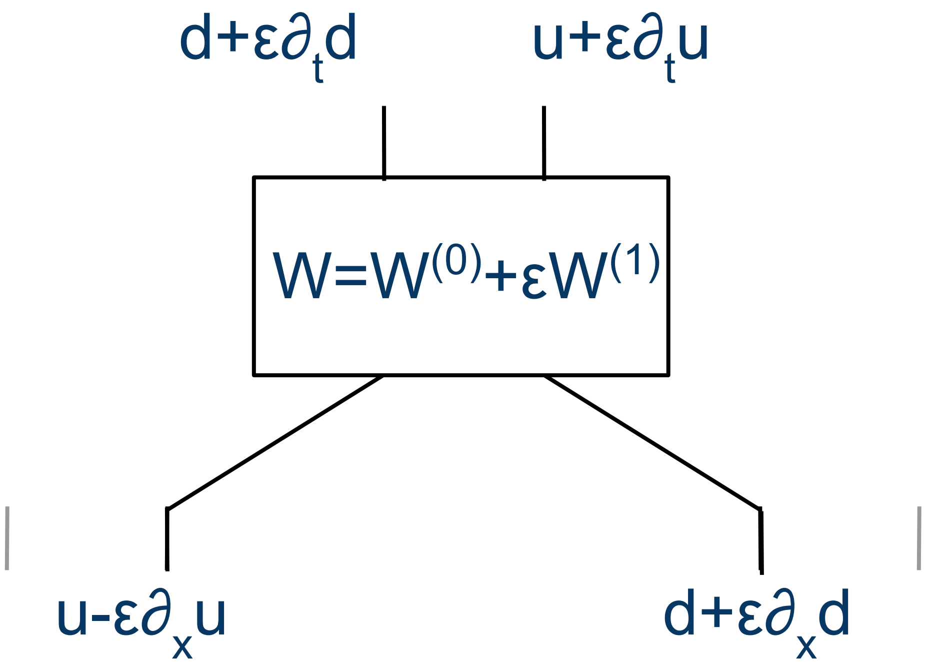
Dirac QW

proof sketch



Dirac QW

proof sketch

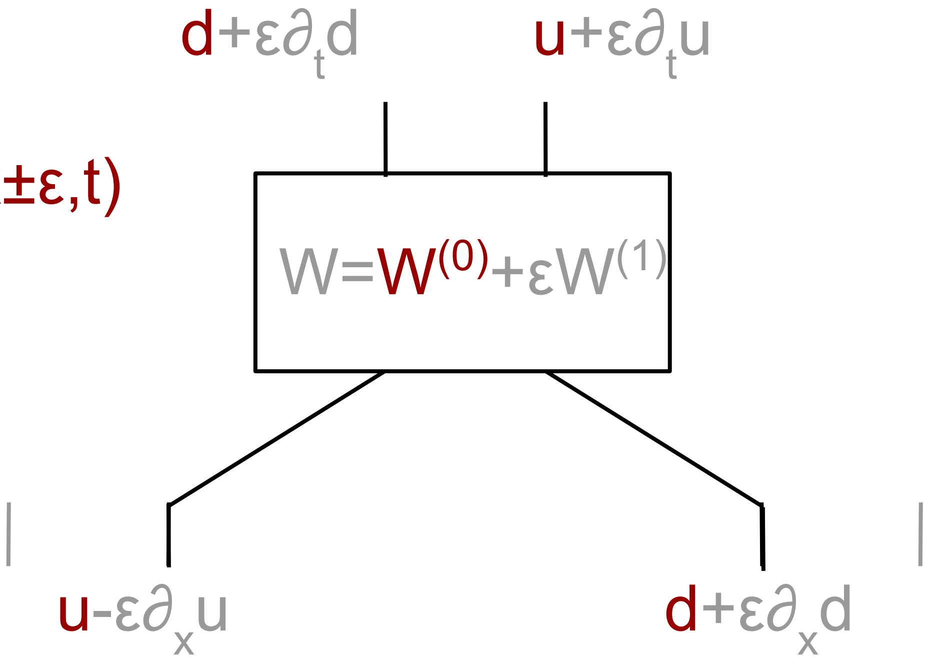


Dirac QW

proof sketch

Order 0

$u(x,t+\varepsilon) \simeq u(x\pm\varepsilon,t)$



Dirac QW

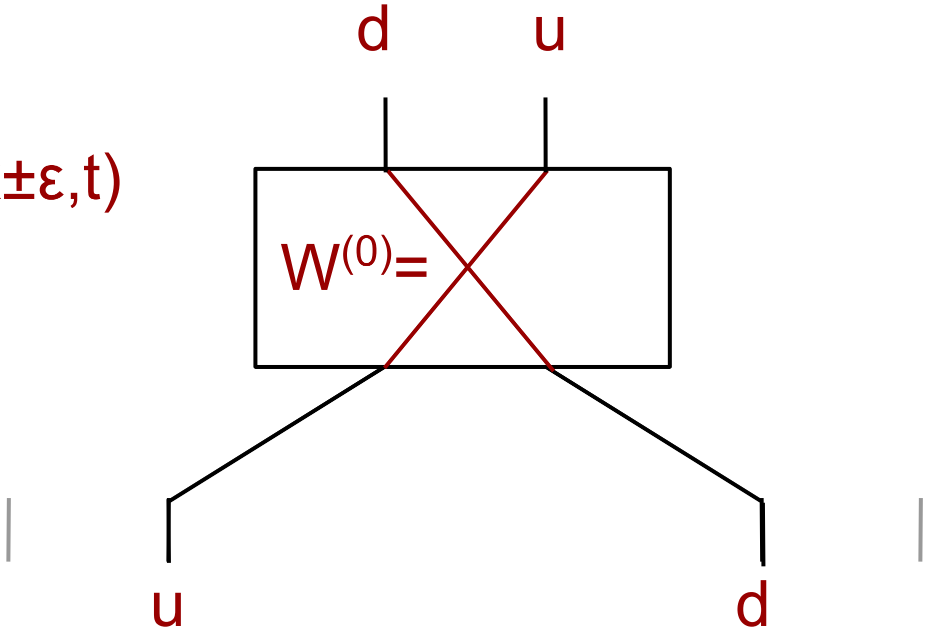
proof sketch

Order 0

$u(x,t+\varepsilon) \simeq u(x\pm\varepsilon,t)$

$\Rightarrow$

$W^{(0)} = X$

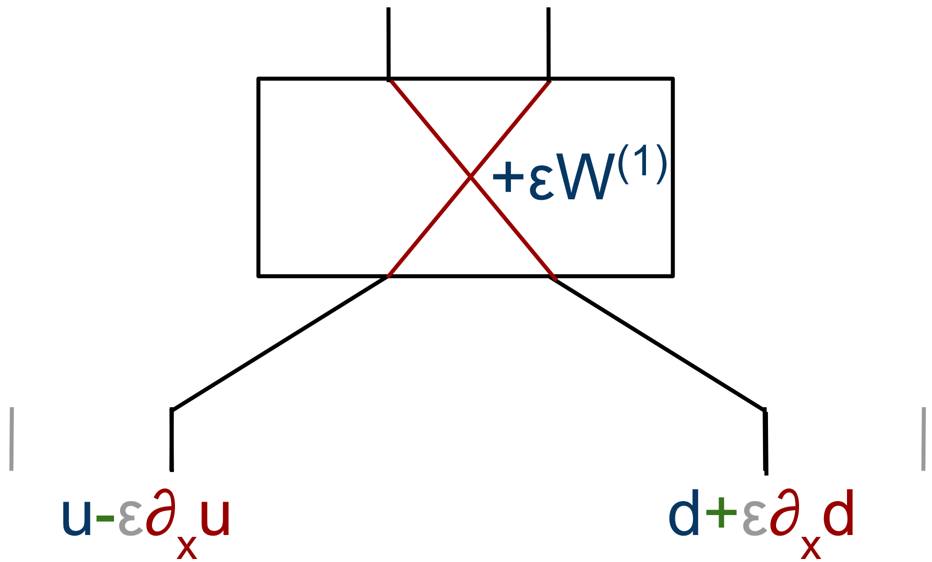


Dirac QW

proof sketch

Order 1

$\partial_t d = \partial_x d + \ldots \qquad \partial_t u = -\partial_x u + \ldots$

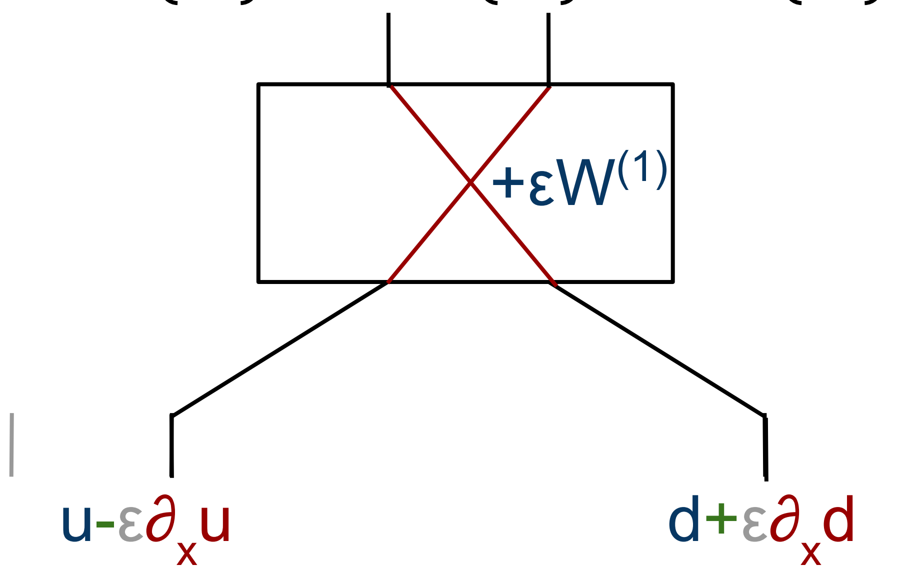


Dirac QW

proof sketch

Order 1

$$\partial_t \begin{pmatrix} u \\ d \end{pmatrix} = -\,\sigma_z \partial_x \begin{pmatrix} u \\ d \end{pmatrix} + \varepsilon W^{(1)} \begin{pmatrix} u \\ d \end{pmatrix}$$



Dirac QW

proof sketch

Order 1

$C = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$

$c = \cos(\theta)$

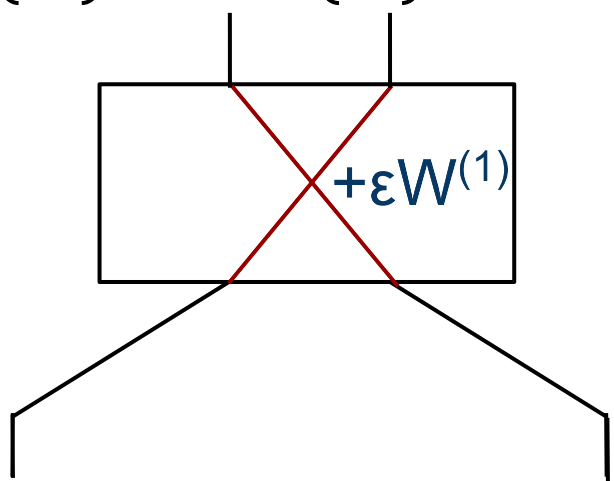
$s = \sin(\theta)$

$\theta = m.\varepsilon$

$C^{(1)} = -im\sigma_y$

$W = CX$

$$\partial_t \begin{pmatrix} u \\ d \end{pmatrix} = -\sigma_z \partial_x \begin{pmatrix} u \\ d \end{pmatrix} + \varepsilon W^{(1)} \begin{pmatrix} u \\ d \end{pmatrix}$$



$u - \varepsilon\partial_x u$ $\quad$ $d + \varepsilon\partial_x d$

$$i\frac{\partial\psi}{\partial t} = -i\sigma_z\frac{\partial\psi}{\partial x} + m\sigma_x\psi$$

Example:

Dirac QW with initial Gaussian wavepacket

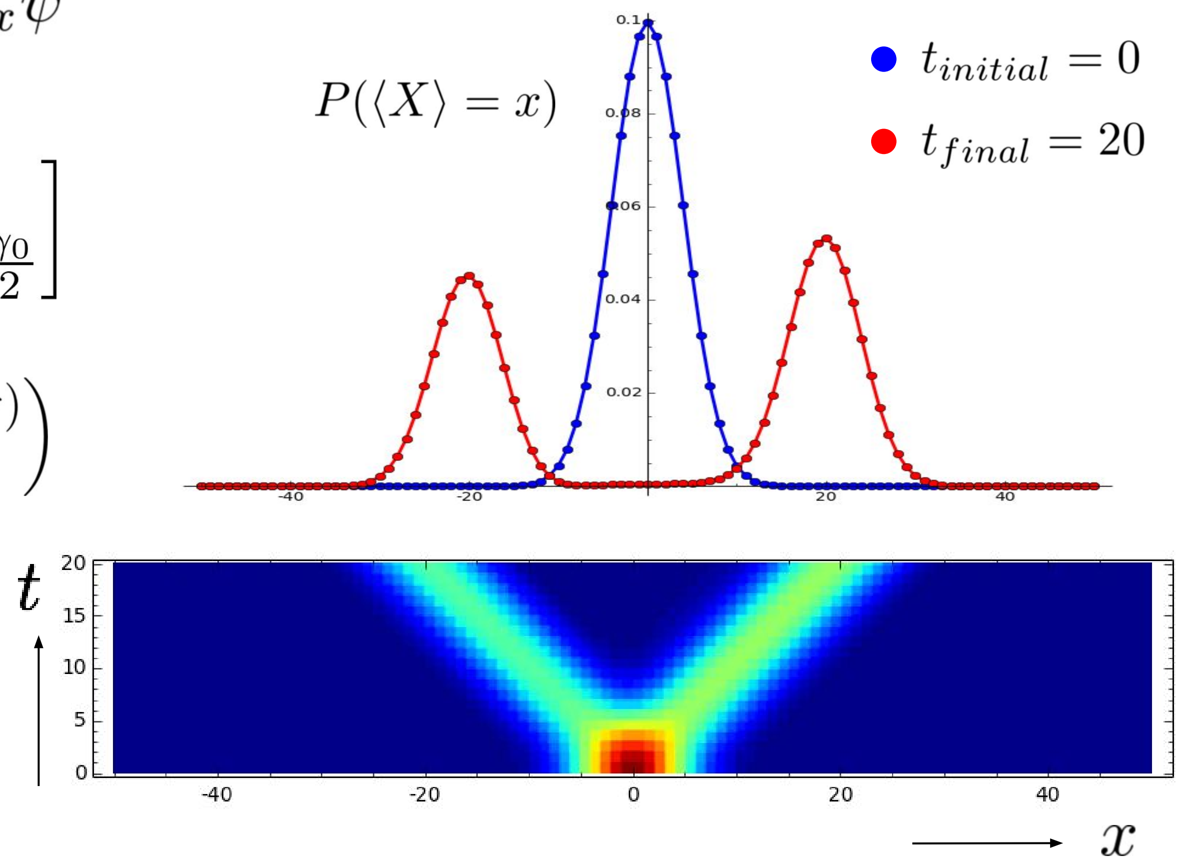
$$i\frac{\partial\psi}{\partial t} = -i\sigma_z\frac{\partial\psi}{\partial x} + m\sigma_x\psi$$

$$\psi_0 \propto \frac{e^{-\frac{(x-x_0)^2}{2\sigma_0^2}}}{\sqrt{2\pi}\sigma_0}\begin{bmatrix}\cos\frac{\gamma_0}{2}\\ e^{i\varphi_0}\sin\frac{\gamma_0}{2}\end{bmatrix}$$

$$C(\varepsilon) = \begin{pmatrix}\cos(m\varepsilon) & -i\sin(m\varepsilon)\\ -i\sin(m\varepsilon) & \cos(m\varepsilon)\end{pmatrix}$$

$m = 1, \varepsilon = 1\times10^{-2}$

$x_0 = 0, \sigma_0 = 4, \gamma_0 = \pi/2, \varphi_0 = \pi/2$

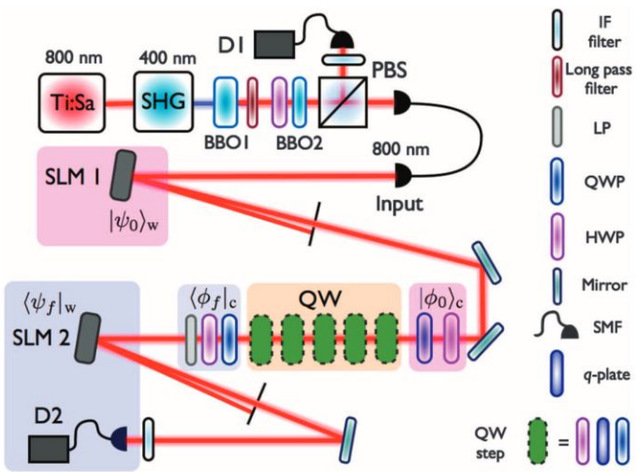


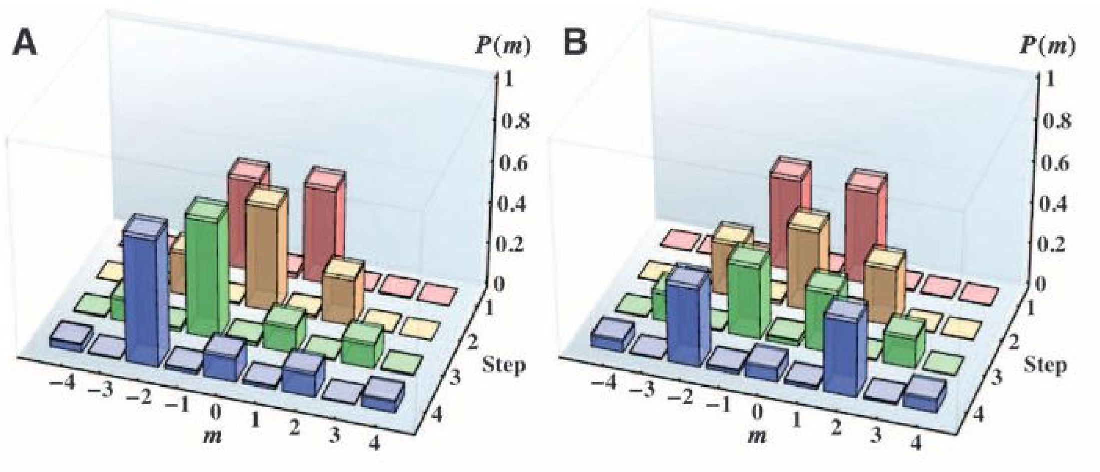
[Arrighi, P., Nesme, V., & MF (2014). The Dirac equation as a quantum walk: higher dimensions, observational convergence. *Journal of Physics A: Mathematical and Theoretical*, *47*(46), 465302.]

QW for quantum simulation

Photonic implementation: QW in the orbital angular momentum space of light

one photon and two simultaneous photons

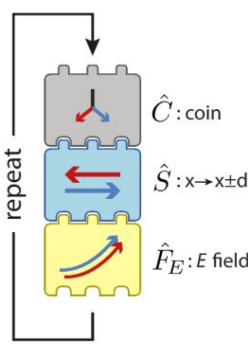


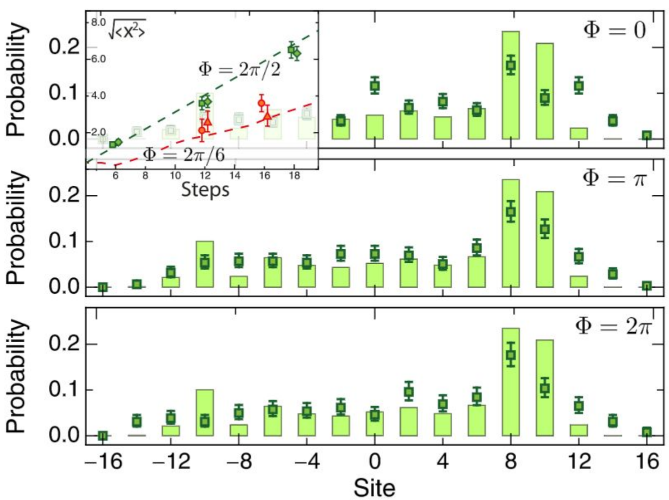


[Cardano, F., Massa, F., Qassim, H., Karimi, E., Slussarenko, S., Paparo, D., ... & Marrucci, L. (2015). Quantum walks and wavepacket dynamics on a lattice with twisted photons. *Science Advances*, *1*(2), e1500087]

QW for quantum simulation

Cold atoms illustrating transport dynamics of *electric* quantum walks



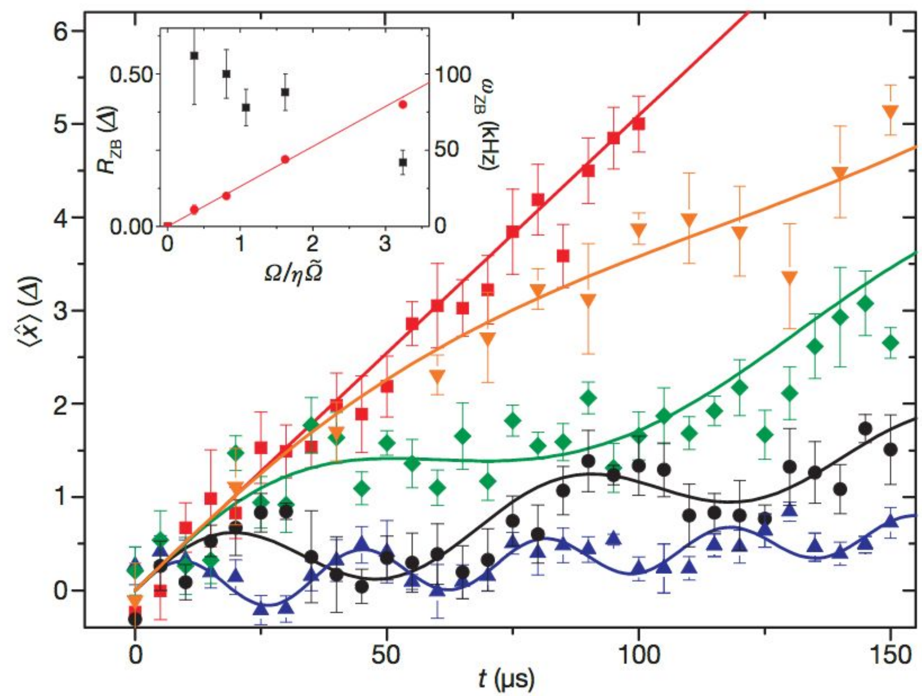


$$\hat{F}_E = e^{i\Phi\hat{x}}$$

$$\Phi = qEd\frac{\tau}{\hbar}$$

[Genske, M., Alt, W., Steffen, A., Werner, A. H., Werner, R. F., Meschede, D., & Alberti, A. (2013). Electric quantum walks with individual atoms. *Physical Review Letters*, *110*(19), 190601]

Single trapped ion behaving as a relativistic quantum particle



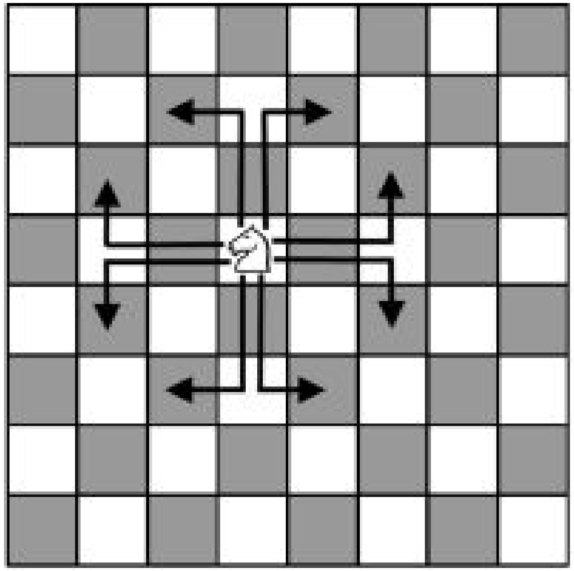
[Gerritsma, R., Kirchmair, G., Zähringer, F., Solano, E., Blatt, R., & Roos, C. F. (2010). Quantum simulation of the Dirac equation. *Nature*, *463*(7277), 68-71]

QW as toy models

Dirac equation vs. chess game:

$$i\partial_0 \psi = D\psi$$

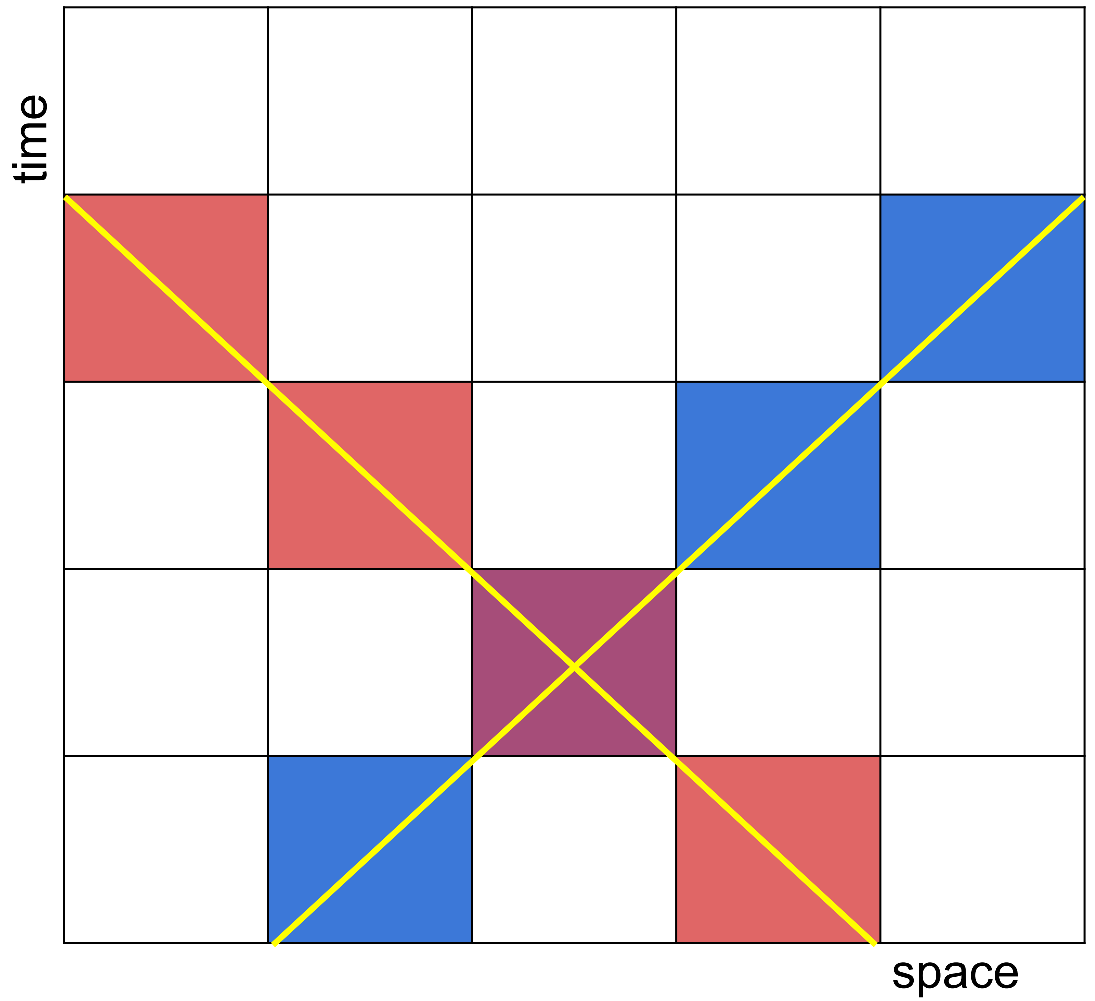
$$D = m\alpha^0 - i\sum_{j=1}^{3} \alpha^j \partial_j$$



See also: [D'Ariano, G. M., & Perinotti, P. (2014). Derivation of the Dirac equation from principles of information processing. *Physical Review A*, *90*(6), 062106]

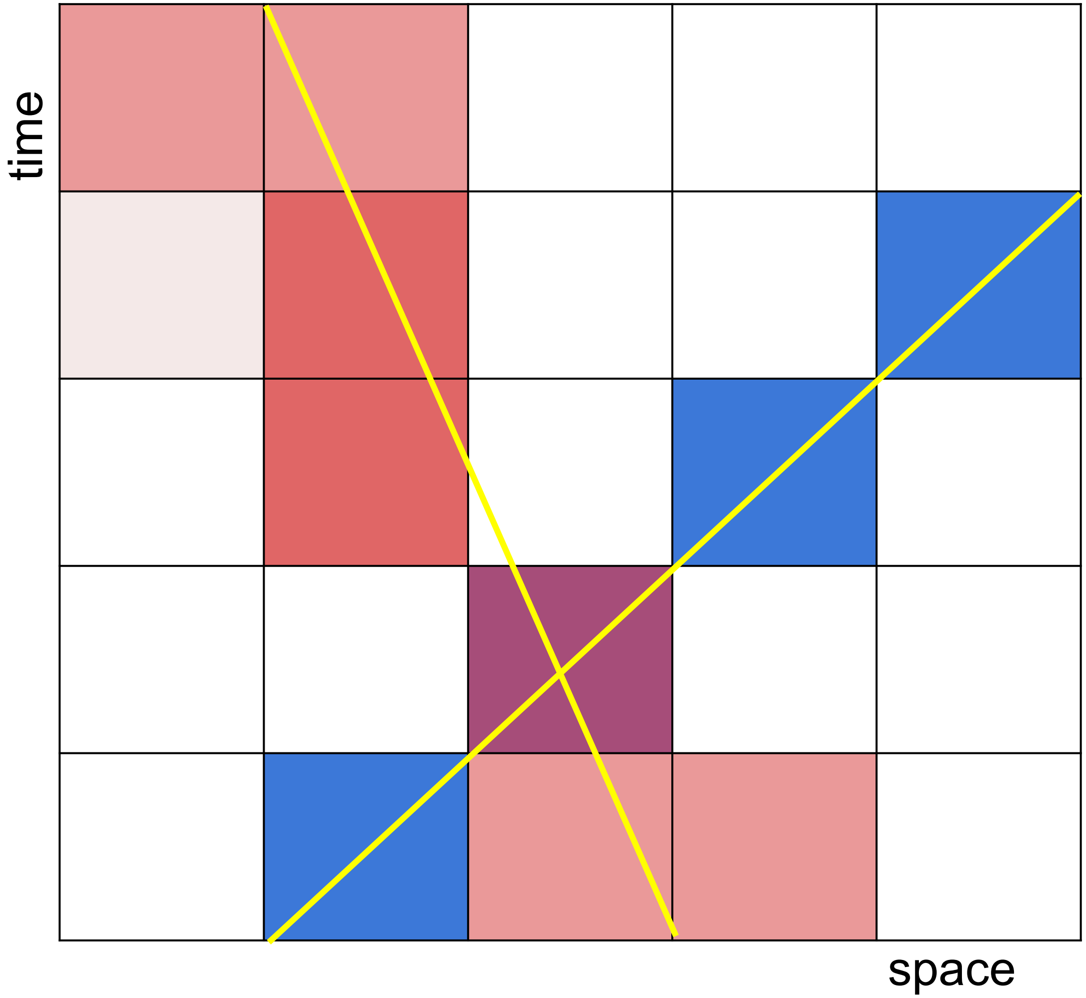
Curved space : problem 1

From

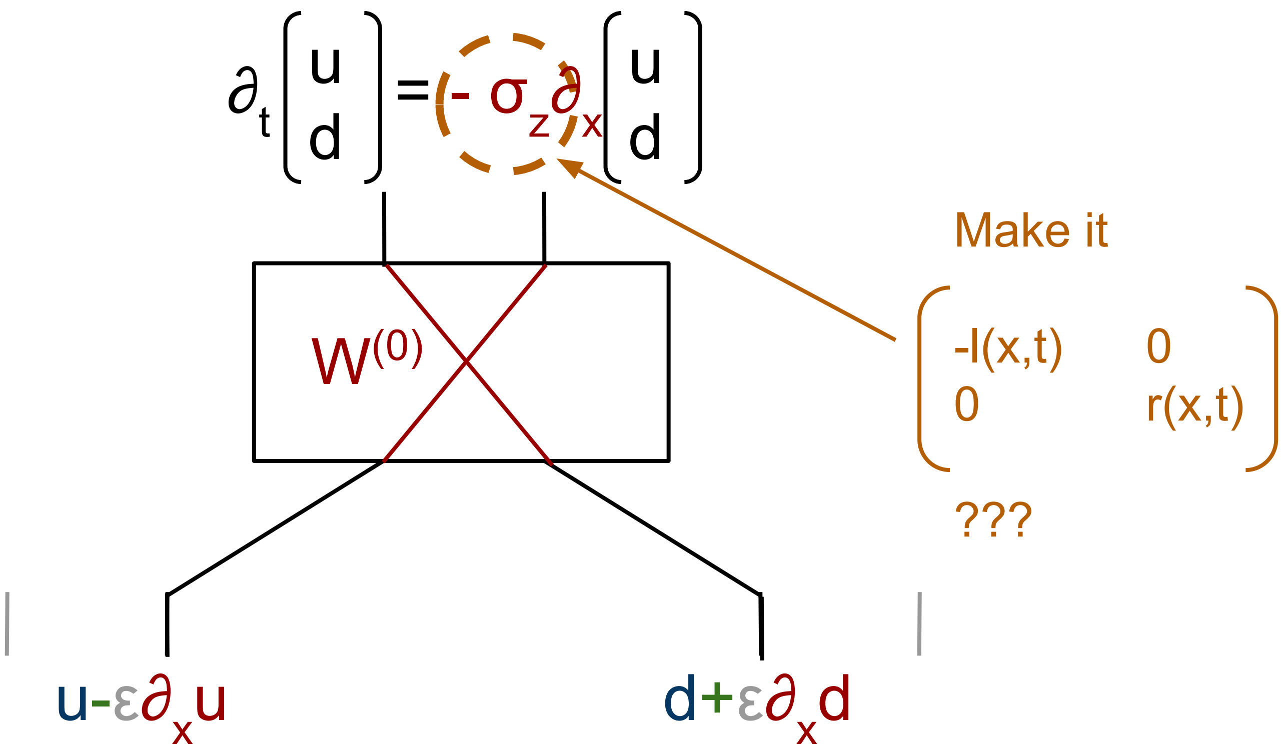


Curved space : problem 1

To



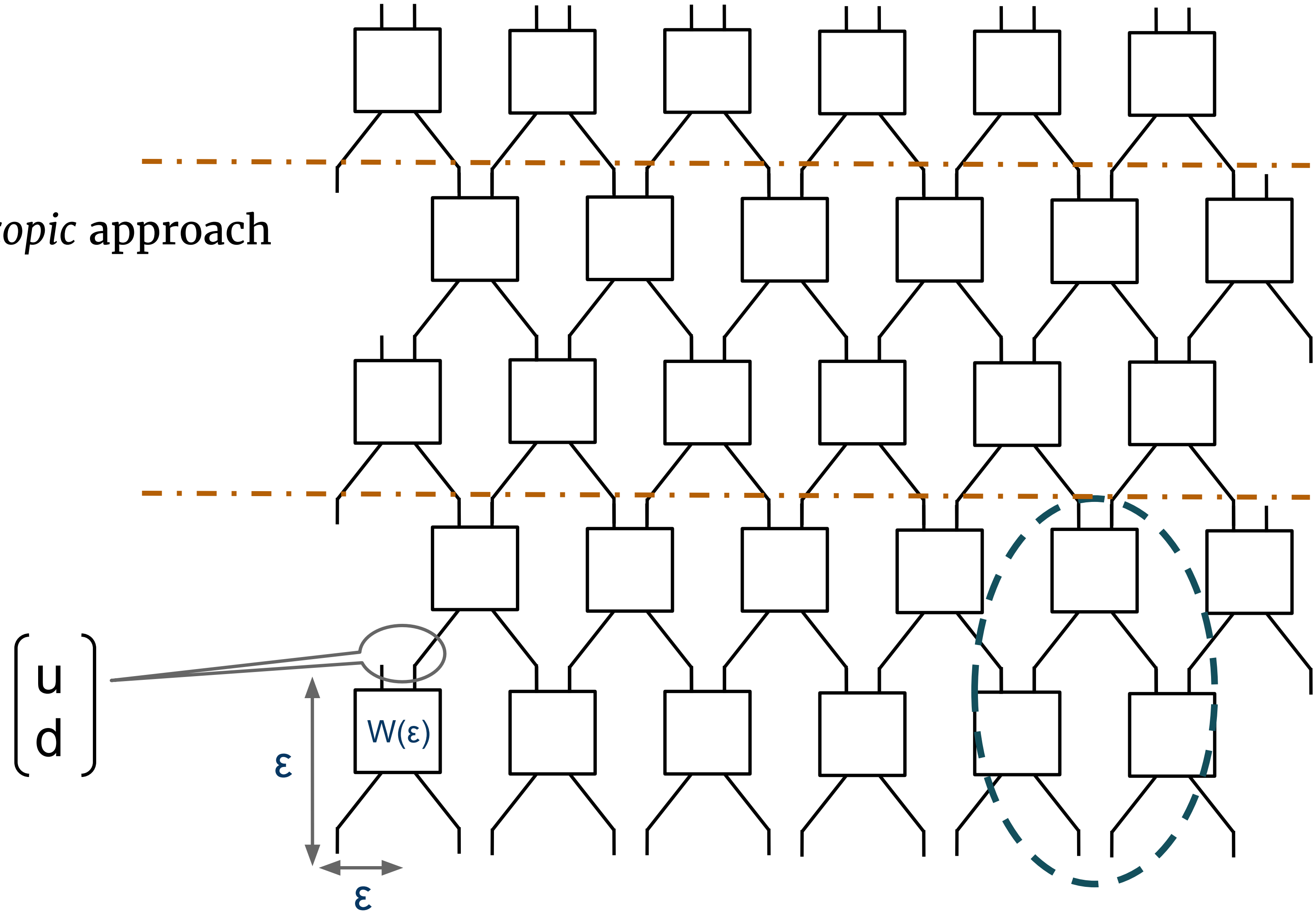
Curved space : problem



Transport term is fixed by 0th order & discrete grid !

Curved space : idea 0

Stroboscopic approach



[Di Molfetta , F. Debbasch, M. E. Brachet, *"Quantum walks as massless Dirac Fermions in curved Space-Time"*, PRA, arXiv:1212.5821]

Curved space : idea 0

Stroboscopic approach

Requires:

$$\partial_t \begin{pmatrix} u \\ d \end{pmatrix} = \left(-\sigma_z \partial_x\right) \begin{pmatrix} u \\ d \end{pmatrix}$$

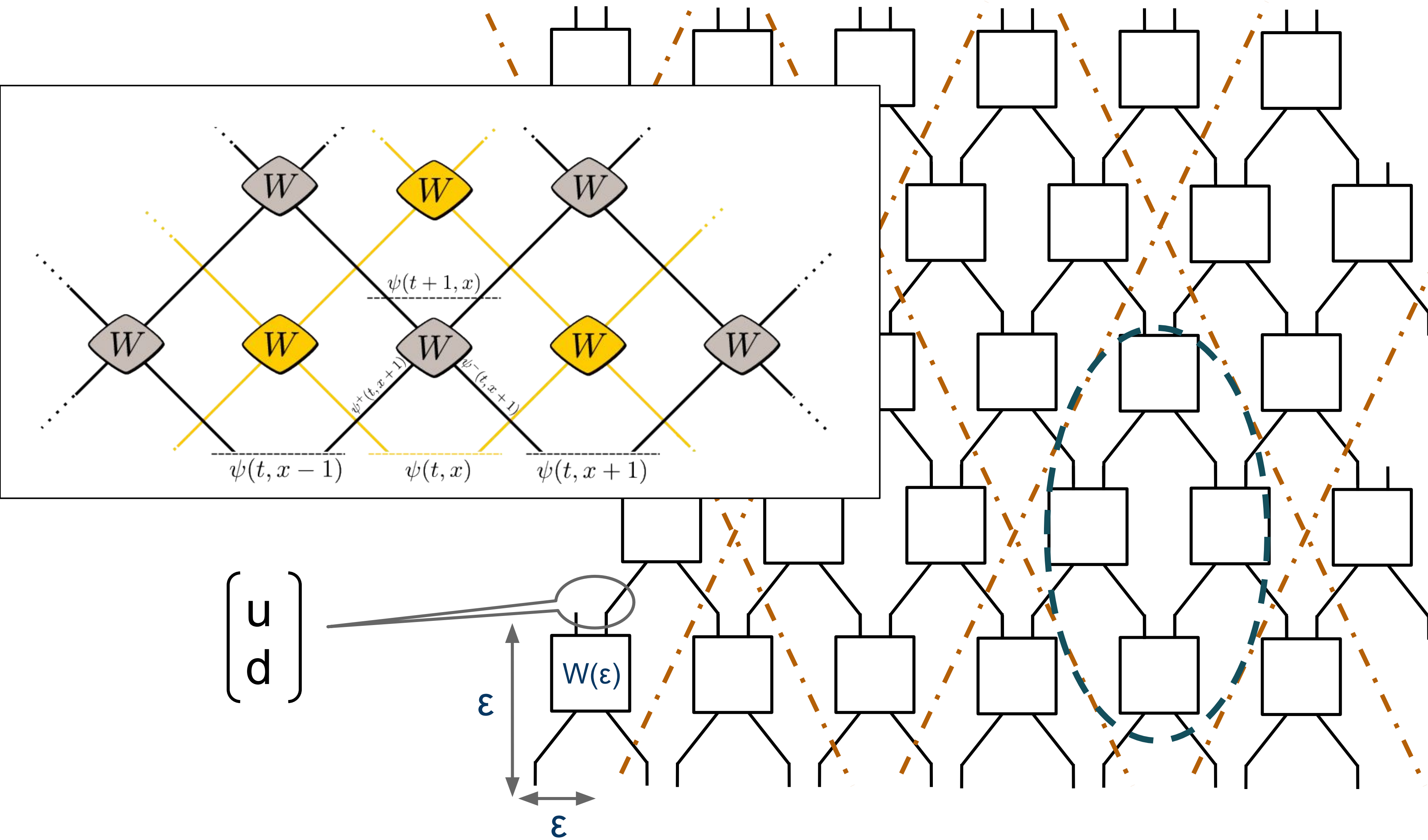
- l = r = c
- massless
- (1+1)

Made it

$$\begin{pmatrix} -c(x,t) & 0 \\ 0 & c(x,t) \end{pmatrix}$$

[Di Molfetta , F. Debbasch, M. E. Brachet, *"Quantum walks as massless Dirac Fermions in curved Space-Time"*, PRA, arXiv:1212.5821]

Curved space : idea 1

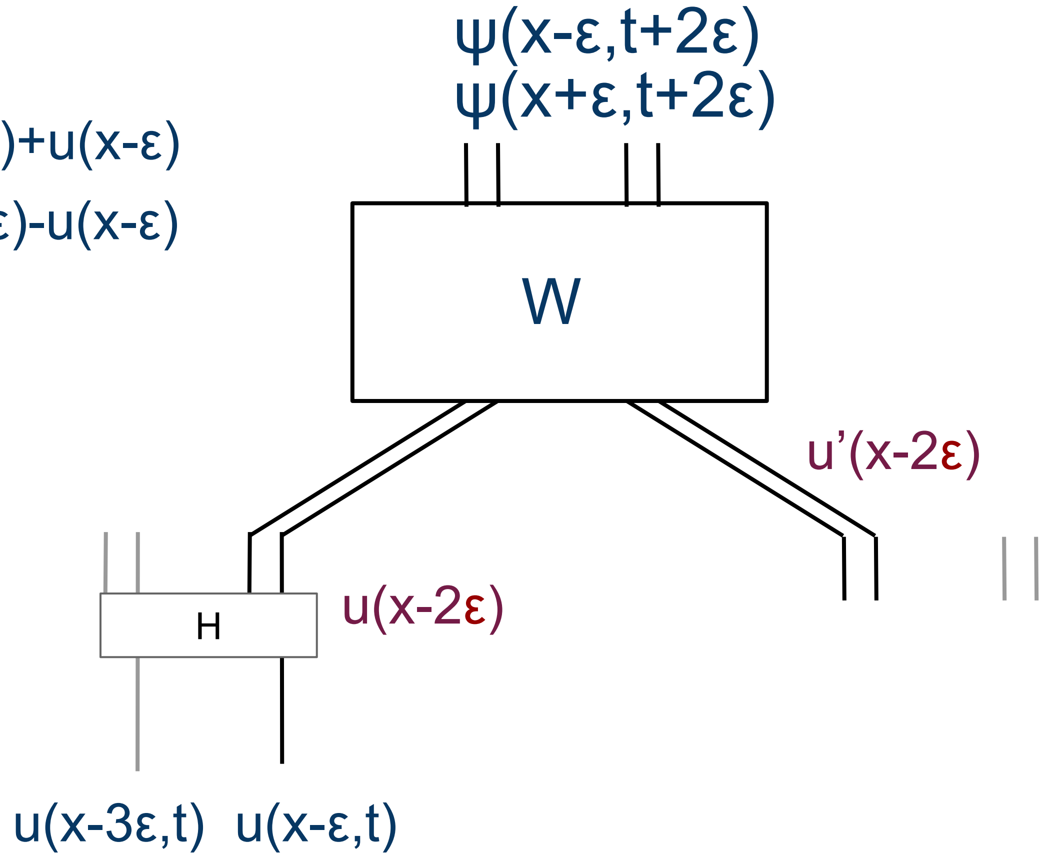


Curved space : idea 1

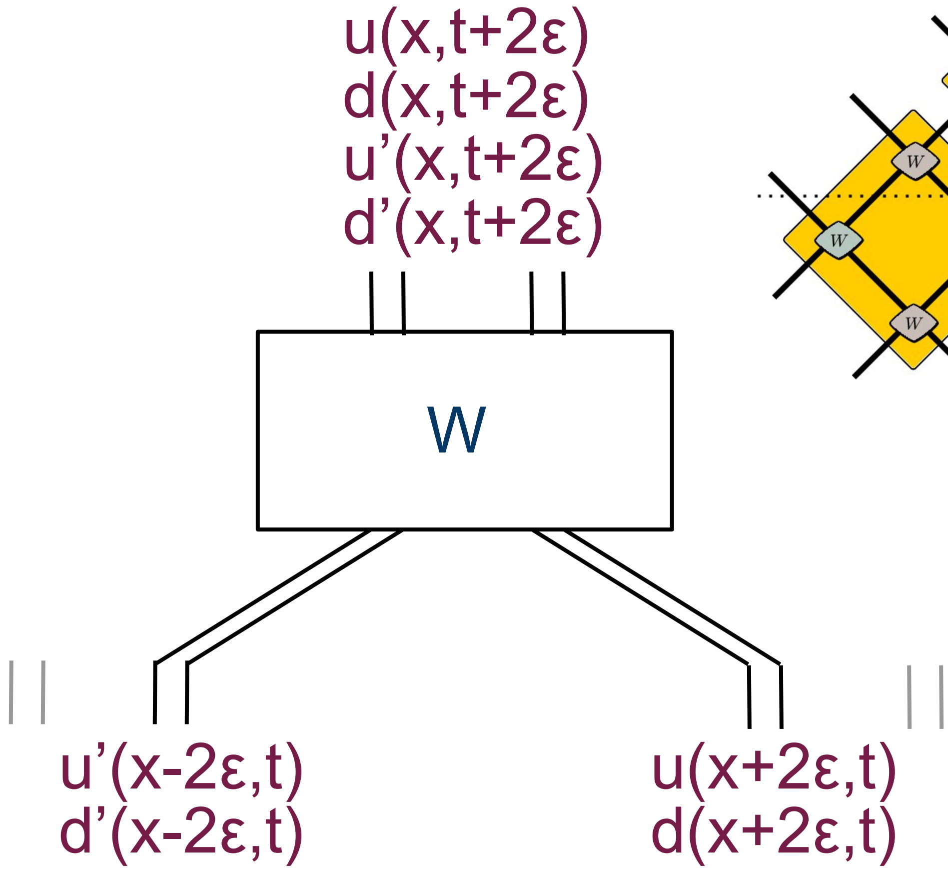
States

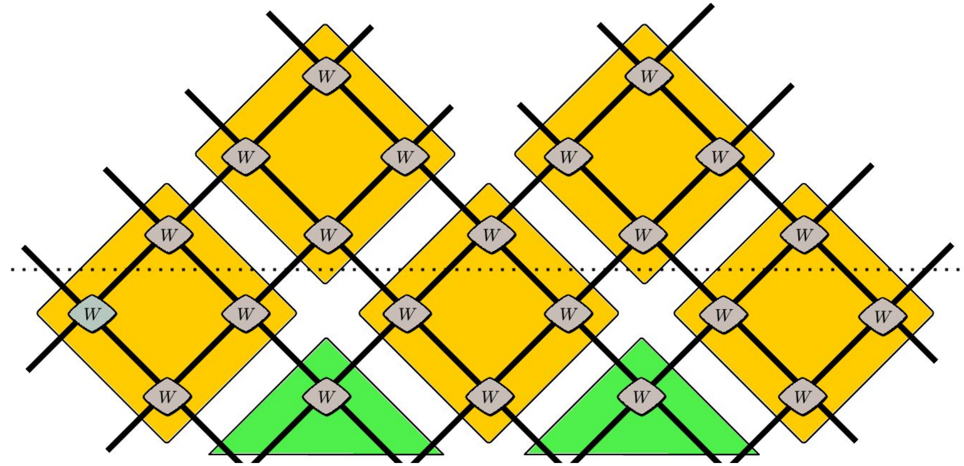
u(x) := u(x+ε)+u(x-ε)

u’(x) := u(x+ε)-u(x-ε)

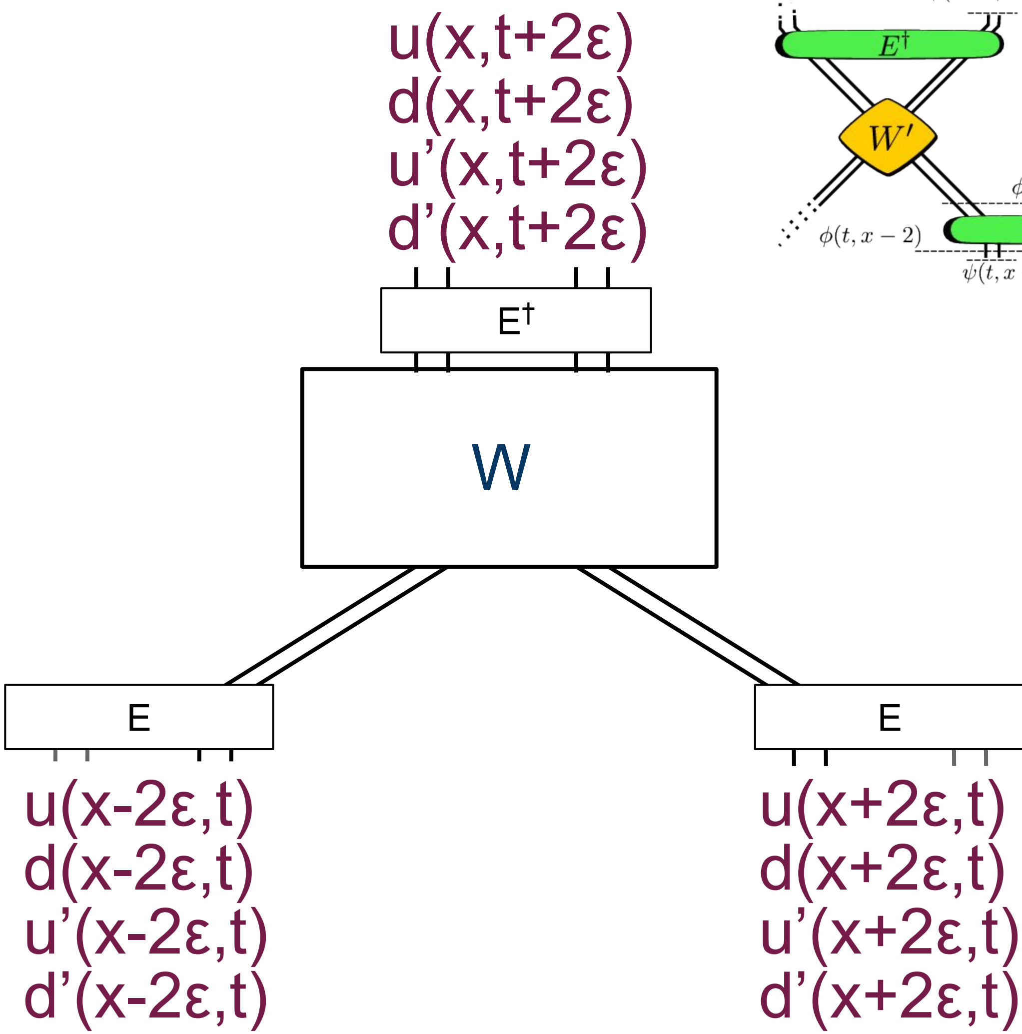


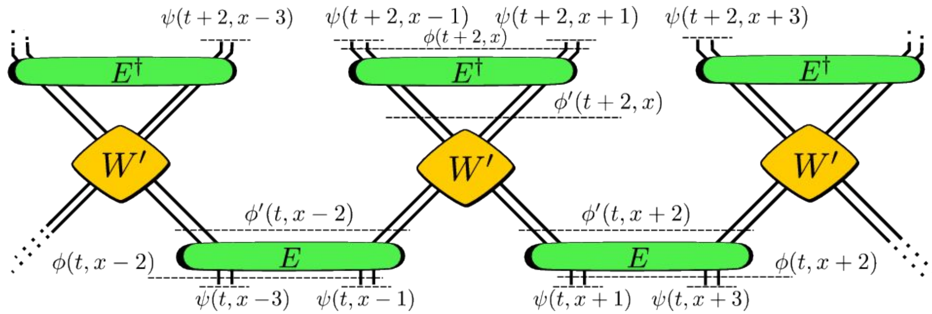
Curved space : idea 1



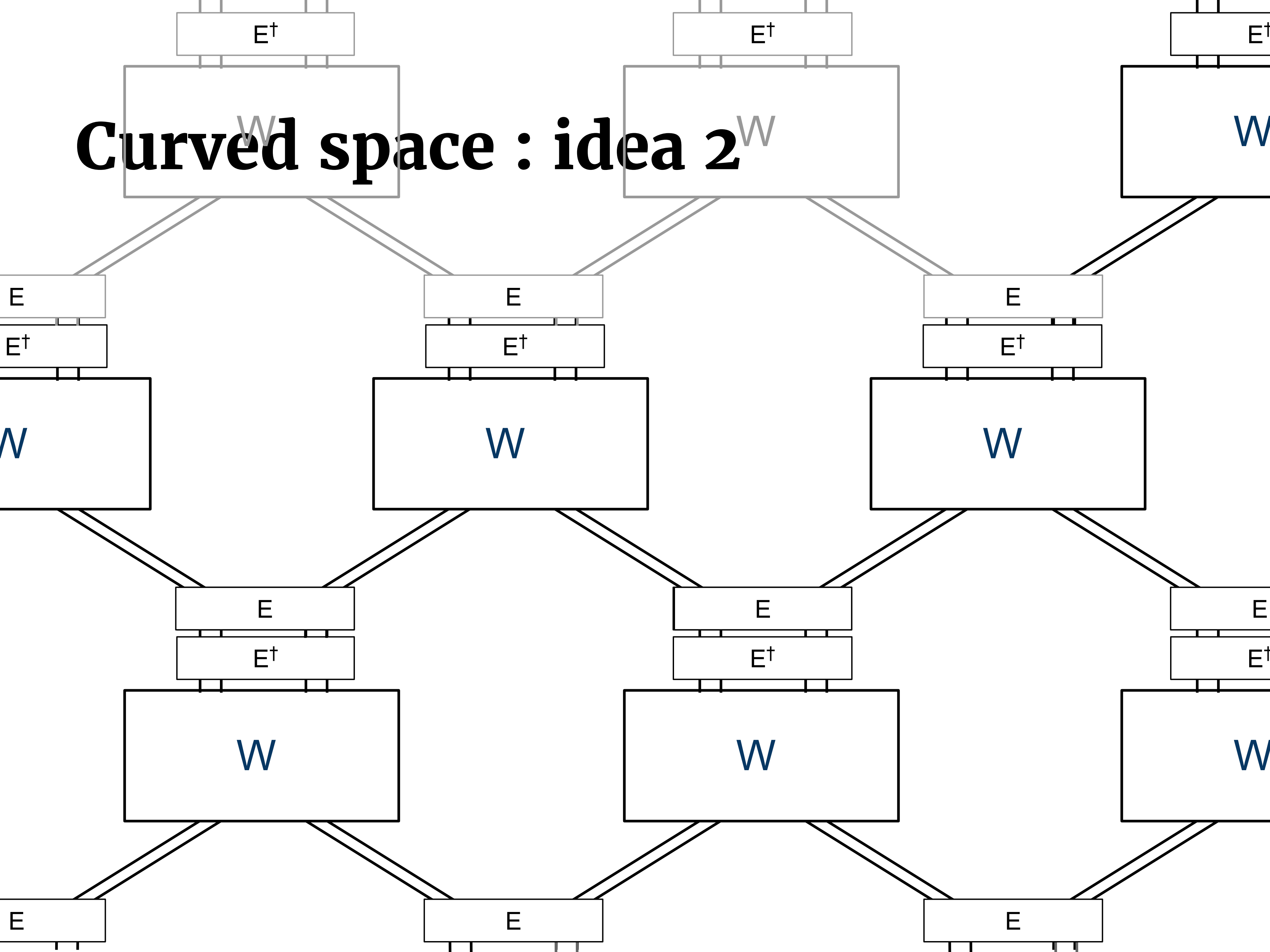


Curved space : idea 2

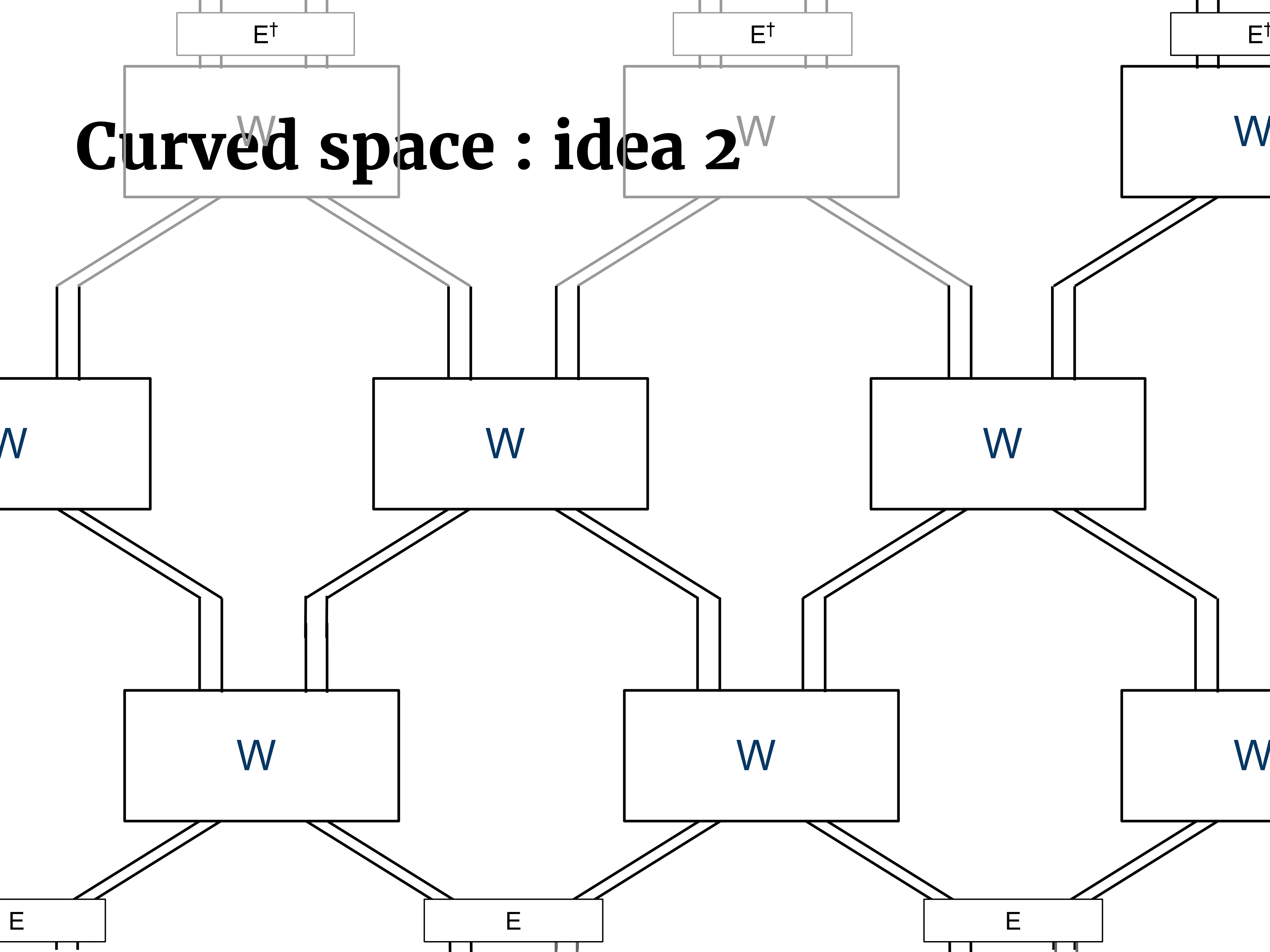




Curved space : idea 2



Curved space : idea 2



Curved space: main result

Theorem : In the continuum limit, this QW converges to the class of PDE's:

$$i\partial_0 \psi = H\psi$$

$$H = i\sum_i (B_1^{(i)}\partial_i + \frac{1}{2}\sum_i \partial_i B_1^{(i)}) - C$$

$$B_1, C \in \mathrm{Her}$$

$$\lambda_i(B_1) \in [-1, 1]$$

$$B_1^{(i)} = -\sum_j \alpha^j \frac{e_j^i}{e_0^0} - e_0^i$$

$$C = -\frac{m}{e_0^0}\beta + \frac{1}{4e_0^0}\gamma_5 \alpha^\mu \varepsilon_{\lambda\kappa\rho\sigma} e^{k\mu} e^{\rho\nu} \partial_\mu e_\nu^\sigma$$

$$\hbar = c = G = 1$$

$$g_{\mu\nu} e_a^\mu e_b^\nu = \eta_{ab}$$

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$$

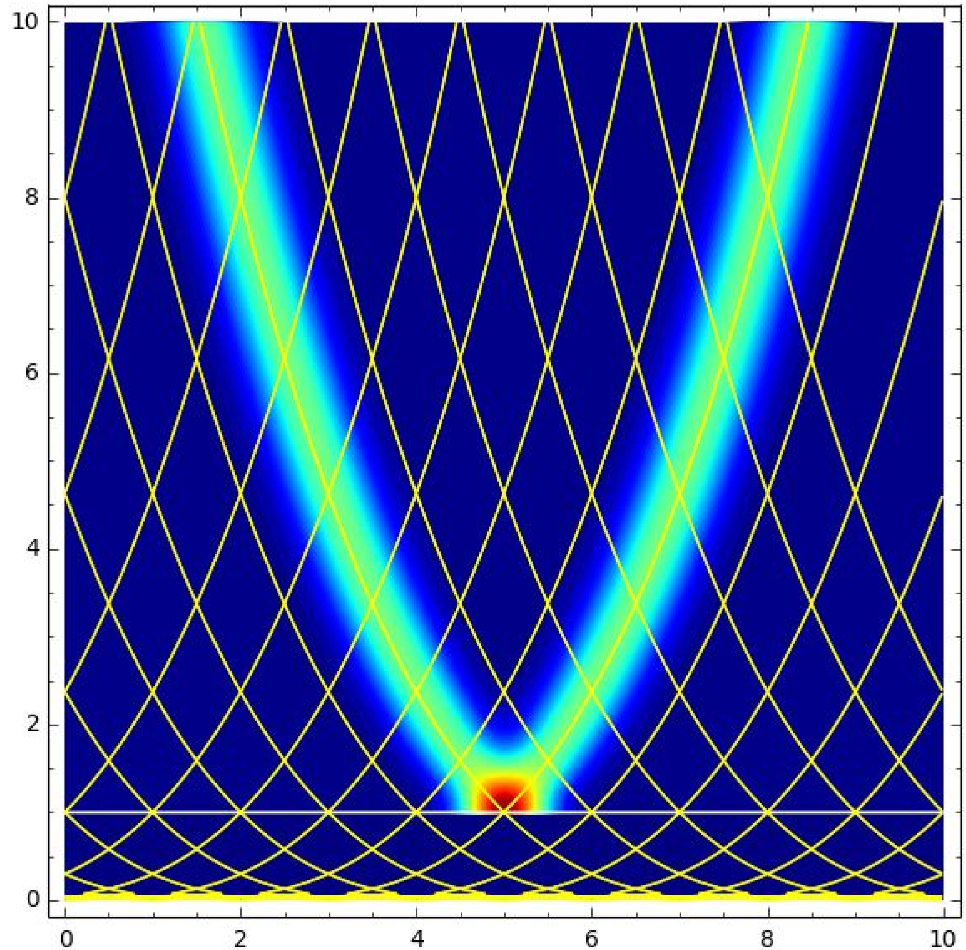
$$\beta = \gamma^0, \alpha^\mu = \gamma^0\gamma^\mu$$

[P. Arrighi, S. Facchini, MF, *"Quantum walking in curved spacetime."* Quantum Information Processing *(2015): 1-20, see also arXiv:1609.00305v2*]

Curved space simulations

Robertson-walker metric

Expanding (or contracting) universe, homogeneous and isotropic.

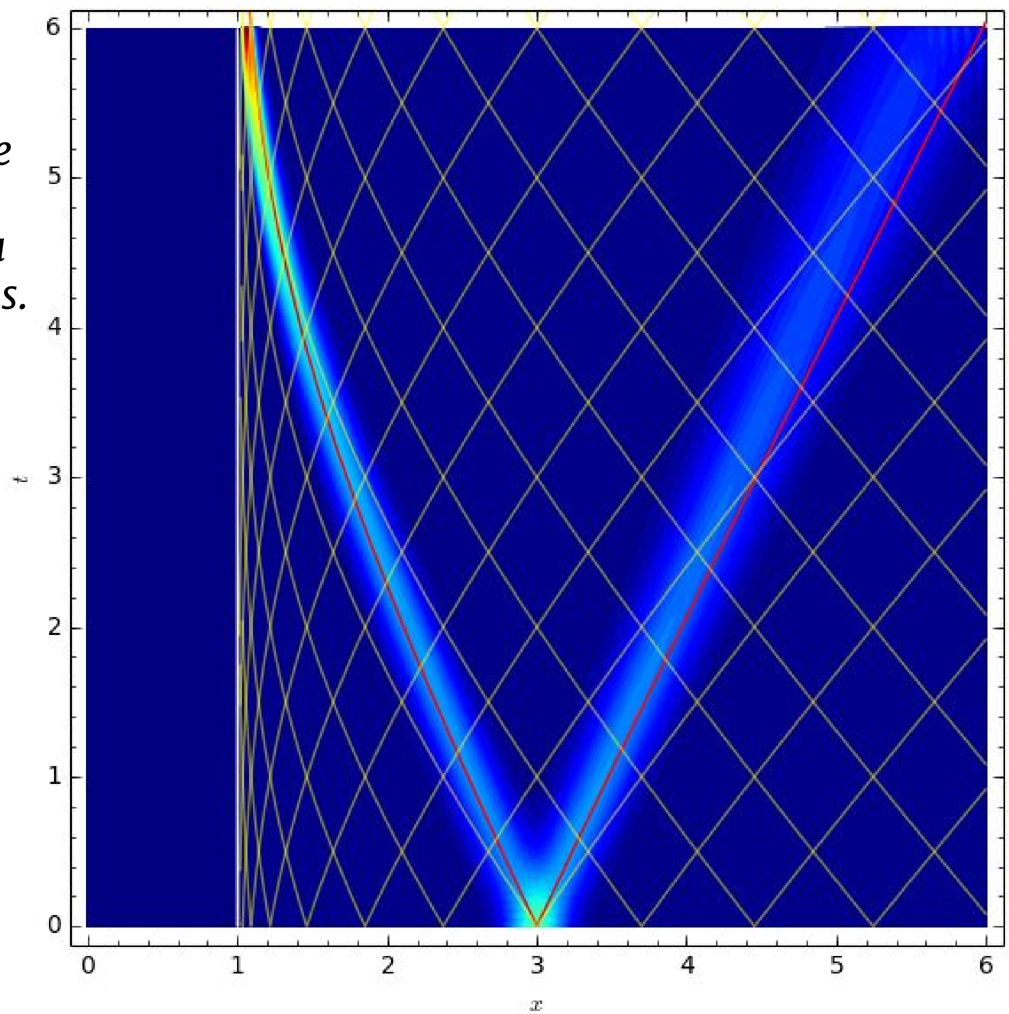


These simulations are delivered as a SageMath worksheet (Python/NumPy/SciPy), and are available at: http://pageperso.lif.univ-mrs.fr/~pablo.arrighi/publis/CurvedSpacetimeDiracEquation.sws

Curved space simulations

Schwarzschild metric

Describing the gravitational field outside a spherical mass.



$$B_1 = -\left(1 - \frac{2M}{x}\right)\sigma_z$$

$$C_1 = -m\left(1 - \frac{2M}{x}\right)^{1/2}\sigma_x$$

Initial state and parameters:

$$\psi(t=0,x) \propto \int (u_+(p) + u_-(p)) \times e^{-(p-p_0)^2/(2\sigma^2)+i(x-x_0)p} dp$$

$$x_0 = 3.0, p_0 = 50, \sigma = 1.56$$

$$m = 50, M = 0.5, \varepsilon = 5 \times 10^{-5}$$

These simulations are delivered as a SageMath worksheet (Python/NumPy/SciPy), and are available at: http://pageperso.lif.univ-mrs.fr/~pablo.arrighi/publis/CurvedSpacetimeDiracEquation.sws

Conclusion

QW for non-interacting particles in curved spacetime:

- quantum simulation of quantum relativistic phenomena.
- to simplify, understand, offer toy models.
- control propagation speed over lattices.

See also

- Discrete Lorentz covariance

Future

- Discrete general covariance. Interactions.
- Active groups: Arrighi AMU, Debbasch UPMC.

[Arrighi et al (2014). Discrete Lorentz covariance for quantum walks and quantum cellular automata. *New Journal of Physics*, *16*(9), 093007]